AP Live Mock Exam #2 – Question 1 (a)-(e)

For what would be accepted as work and answers for the actual AP Exam, please watch: https://bit.ly/3dqwuh3

(a) $g'(5)$ is the slope of the tangent line to the graph of g at $x = 5$. $g'(5) = -\frac{5}{3}$

(b) $b'(x) = 2x^2 g'(x) + 4x\, g(x)$.

$$b'(5) = 2(5)^2 g'(5) + 4(5)g(5) = 50\left(-\frac{5}{3}\right) + 20(1) = \frac{-190}{3} \approx -63.333.$$

(c) $w'(x) = \dfrac{(3h'(x) - 1)(2x + 1) - 2(3h(x) - x)}{(2x + 1)^2}$

$$w'(5) = \frac{(3h'(5) - 1)(2(5) + 1) - 2(3h(5) - 5)}{(2(5) + 1)^2} = \frac{\left(3\left(-\frac{5}{3}\right) - 1\right)(11) - 2(3(1) - 5)}{(11)^2} = -\frac{62}{121}$$

≈ -0.512

(d) $M(x) = \dfrac{d}{dx}\left[\int_0^{2x} g(t)\,dt\right] = g(2x) \cdot 2 = 2g(2x)$.

$M'(x) = 2g'(2x) \cdot 2 = 4g'(2x)$. $\quad M'(2.5) = 4g'\big(2(2.5)\big) = 4g'(5) = 4\left(-\frac{5}{3}\right) = -\frac{20}{3}$

(e) $M'(c) = \dfrac{M(b) - M(a)}{b - a}$;

$$M'(2.5) = \frac{M(4) - M(1)}{4 - 1} = \frac{\big(2g(2(4)) - 2g(2(1))\big)}{3} = \frac{2}{3}\big(g(8) - g(2)\big)$$

$$4g'\big(2(2.5)\big) = \frac{2}{3}\big(g(8) - g(2)\big)$$

$$g(8) - g(2) = \frac{3}{2}\left(4\left(-\frac{5}{3}\right)\right) = -10$$

AP Live Mock Exam #2 – Question 1 (f)-(g)

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(f) Because g is differentiable, g is continuous so, $\lim_{x\to 5} g(x) = g(5) = 1$.

Also, $\lim_{x\to 5} g(x) = \lim_{x\to 5} \dfrac{x + 5\cos\left(\frac{1}{5}\pi x\right)}{3 - \sqrt{f(x)}}$, so $\lim_{x\to 5} \dfrac{x + 5\cos\left(\frac{1}{5}\pi x\right)}{3 - \sqrt{f(x)}} = 1$

Because $\lim_{x\to 5}\left(x + 5\cos\left(\frac{1}{5}\pi x\right)\right) = 5 - 5 = 0$, we must also have $\lim_{x\to 5}\left(3 - \sqrt{f(x)}\right) = 0$.

Thus $\lim_{x\to 5} f(x) = 9$. Because f is differentiable, f is continuous, so $f(5) = \lim_{x\to 5} f(x) = 9$.

Also, because f is twice differentiable, f' is continuous, so $\lim_{x\to 5} f'(x) = f'(5)$ exists.

Using L'Hospital's Rule,

$$\lim_{x\to 5} \frac{x + 5\cos\left(\frac{1}{5}\pi x\right)}{3 - \sqrt{f(x)}} = \lim_{x\to 5} \frac{1 - \sin\left(\frac{1}{5}\pi x\right)}{-\frac{1}{2\sqrt{f(x)}} f'(x)} = \frac{1 - \sin\left(\frac{1}{5}\pi 5\right)}{-\frac{1}{2\sqrt{f(5)}} f'(5)} = \frac{1 - 0}{-\frac{1}{2\sqrt{9}} f'(5)} = 1$$

Thus $f'(5) = -6$.

(g) Because h and g are differentiable, h and g are continuous, so
$\lim_{x\to 5} h(x) = h(5) = 1$ and $\lim_{x\to 5} g(x) = g(5) = 1$.

Because $h(x) \le k(x) \le g(x)$ for $4 < x < 6$, it follows from the squeeze theorem that
$1 = \lim_{x\to 5} h(x) \le \lim_{x\to 5} k(x) \le \lim_{x\to 5} g(x) = 1$ and $\lim_{x\to 5} k(x) = 1$.

Also, $1 = h(5) \le k(5) \le g(5) = 1$, so $k(5) = 1$.

Thus k is continuous at $x = 5$.

AP Live Mock Exam #2 – Question 2

For what would be accepted as work and answers for the actual AP Exam, please watch: https://bit.ly/3dqwuh3

(a) $f'(x) = \pi\cos(\pi x) - \dfrac{1}{2-x}$

$f'(1) = \pi\cos(\pi) - 1 = -\pi - 1$

(b)

$$k'(x) = h'(f(x)+2)\cdot f'(x)$$

$$k'(1) = h'(f(1)+2)\cdot f'(1) = h'(\sin(\pi) + \ln(2-1) + 2)\cdot f'(1) = h'(2)\cdot f'(1)$$

$$= \left(-\frac{1}{3}\right)(-\pi - 1) = \frac{\pi+1}{3} \approx 1.380 \text{ or } 1.381$$

(c) $\displaystyle\int_{-5}^{-1} g'(x)dx = g(x)\Big|_{-5}^{-1} = g(-1) - g(-5) = 1 - 10 = -9.$

(d) $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\left(h\left(-1+\frac{5k}{n}\right)\right)\frac{5}{n} = \int_{-1}^{4} h(x)\,dx = \frac{1}{2} - \frac{3}{2} - \frac{1}{4} + \frac{1}{4} = -1$

(e) Horizontal tangents will occur when $g'(x) = 0$. Since g is twice differentiable, g' is continuous and the Intermediate Value Theorem can be applied to $g'(x)$ on the interval (-5, 0).

For $-4 < x < -3,\ g'(-4) = -1 < 0 < 4 = g'(-3)$
and for $-2 < x < -1,\ g'(-2) = 1 > 0 > -2 = g'(-1)$.

Thus $g'(x) = 0$ on both the interval $-4 < x < -3$ and $-2 < x < -1$.

Therefore $g(x)$has at least two horizontal tangents on the interval $-5 < x < 0$.